Indian J Phys 80 (11), 1071-1073 (2006)

IJP

Holography and hydrodynamics

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Abstract We describe some recent insights from string theory into holography, especially in the context of a hydrodynamic ... ion of black hole physics

Keywords Quark gluon plasma, *AdS/CFT* conjecture, black holes, holography

PACS Nos. 11 15 Me, 11 10 Wx, 12 38 Mh, 04 70 Dy, 47 10 -q

1. Introduction

...ty seems to stand apart from all the other forces of ...re in many different ways Whereas the quantum ...amics of the other forces are beautifully captured ...in the framework of local quantum field theory, gravity ...resisted all such attempts to tame it. Partly, the reason ...s from Einstein's observation that gravity is woven ... the very fabric of spacetime. This suggests that ...how the dynamics of gravity might be radically ...ferent from that of other forces

The idea that gravity may be *holographic* is one ...ing way in which this difference may manifest itself ... notion was proposed by G't Hooft [1] and elaborated ...y Susskind [2] in the early nineties to make sense of ...me aspects of black holes It remained a tantalising idea ...til a very concrete realisation was proposed by ...aldacena, for a theory of gravity in an asymptotically ...-de Sitter (*AdS*) spacetime. The ramifications of the ...aldacena conjecture for the holographic nature of gravity ...ntinue to be unearthed and we seem to be uncovering ...ery beautiful picture

In this article I will try and briefly sketch a few of the ...portant notions that have emerged in the last few years ...e will focus on some aspects of black holes that are ...lographically reflected in the dynamics of strongly ...teracting thermal field theories In particular, we will describe how the *hydrodynamic* behaviour of such gauge theories is encoded in properties of black hole horizons. Not only has this led to insights into the physics of field theories but it is also giving a new perspective on the membrane paradigm for black holes

AKR, I believe, would have been very happy to see some of these developments since they provide fresh illumination on some of the concepts that AKR himself helped develop This article is dedicated to his memory

2. Holography

The universal Bekenstein-Hawking expression for the entropy of black holes

$$S_{BH} = \frac{A_H}{4G_N} \tag{1}$$

was one of the first clues to the holographic nature of gravity Recall that A_H is the *area* of the horizon and G_N is Newtons' constant The area dependence indicates that any microscopic accounting of this entropy in terms of the number of degrees of freedom of some underlying theory must have an unusual property. It cannot be extensive in the volume In fact, the area dependence seems to say that the degrees of freedom must be more like those of a local field theory in one lower dimension The bold proposal of 't Hooft was that the ***entire gravitational dynamics in d***

spacetime dimensions is completely encoded in a local theory in $(d - 1)$ ***dimensions*** Thus the origin of the term "holography"

The 1997 AdS/CFT conjecture of Maldacena [3] offered the opportunity to flesh out this proposal The conjecture arose from developments in string theory and describes an exact duality between closed superstring theory in the ten dimensional $AdS_5 \times S^5$ spacetime and the maximally supersymmetric Yang-Mills theory in four dimensions The conjecture has sinced been generalised and is now understood to be valid for a large class of asymptotically AdS_5 spacetimes In all cases, a semiclassical gravitational description of the spacetime dynamics is captured by the strong interaction dynamics of a 4d field theory

The connection between dynamics in five dimensional AdS spacetime and four dimensional field theory is a reflection of the holographic nature of this duality In fact, the 4d spacetime in which the field theory lives is the asymptotic boundary of the AdS_5 spacetime There is then a precise dictionary [4,5] between observables in the gauge theory on the ***boundary*** and those of the gravity theory in the ***bulk*** In the semiclassical limit for gravity, this dictionary takes the form

$$\int [D\phi] e^{S_{grav}[\phi]} = \int [D\phi] e^{S_{gauge}[A] \int d^4x \phi_0(x) \mathcal{O}} \qquad (2)$$

Here the left hand side is the gravity side in which we perform an integral over bulk fields ϕ (including the metric) taking specified values ϕ_0 on the boundary The right hand side is the gauge theory in which we have the path integral over gauge fields A. The boundary value (ϕ_0 acts as a source term for a gauge invariant operator $\mathcal{O}_\phi$ of the field theory Thus the gravity theory may be viewed as the generating functional for correlators in the boundary gauge theory

What does this holographic description tell us about black holes, the original motivation of 'tHooft and Susskind? One finds that the Schwarzschild black hole in AdS_5, for instance, is described by the same gauge theory but now at a finite temperature, which is the Hawking temperature of the black hole [6] Thus we can translate questions about black hole physics (in five dimensions) into questions in the strongly interacting dynamics of hot gauge theories (in four dimensions) and *vice versa*!

For instance, the equilibrium thermodynamics of the gauge theory describes black hole thermodynamics In particular, the Bekenstein-Hawking entropy of black holes is the same as the entropy of the thermal gauge the… in the deconfined phase The latter, being extensive in t… three spatial dimensions is actually proportional … area of the horizon of the five dimensional black h… (which is a three dimensional sphere) Other thermodynar… features such as phase transitions in the gauge theory … also have their reflection in the gravity side

This connection to strongly interacting gauge theor… took on a new significance because of recent experime… results from the Relativistic Heavy Ion Collider (RHIC … Brookhaven In these experiments, the high … collisions of gold nuclei seem to have created a … temperature phase of deconfined strongly interacting qua… and gluons (see [7,8] for overviews) This Stron… interacting Quark-Gluon Plasma (SQGP) is very diffic… describe using the tools of perturbative gauge theory… appears as if the dual description in terms … gravitational physics of five dimensional AdS black ho… is better at reproducing some of the features that the RH… data exhibit!

3. Hydrodynamics

One of the challenges of understanding the RHIC … was in describing phenomena which go somewhat bey… equilibrium dynamics In fact, the process of formation … evolution of the SQGP phase are best descri… hydrodynamically Remarkably, it appears that the SQ… behaves almost like a perfect fluid To a … approximation, it is as if the shear viscosity η is neglig… The dimensionless parameter

$$\frac{\eta}{s} = \frac{1}{10}$$

where s is the entropy per unit volume If this descrip… of the data is indeed correct then this would be the n… perfect fluid observed in nature

Such a small value for the shear viscosity is diffi… to explain using perturbative calculations in the ga… theory These give a behaviour

$$s \quad g_{YM}^4 \ln \frac{1}{g_{YM}^2}$$

for small values of the gauge coupling g_{YM}. In th… calculations, the shear viscosity is extracted from the l… frequency pole in the two point function of compone… of the energy momentum tensor

Using the *AdS/CFT* dictionary for gauge theory correlators mentioned in the previous section, this two point function has a gravity interpretation in the five dimensional black hole geometry. It can be related to the absorption cross section for polarised gravitons by the black hole [9]. This latter calculation yields a simple prediction for the shear viscosity at very strong coupling

$$\frac{\eta}{s} = \frac{1}{4\pi} \tag{5}$$

Remarkably, this is very close to the value that best fits data.

In fact, there seems to be something universal about this value of the viscosity extracted from gravity calculations. For a large class of black holes, the absorption calculations yield this same value, somewhat like the universality of the area formula for the entropy. It has been conjectured that the above value for η/s is a universal lower bound. See [10] for an overview and references to the literature on these topics.

Viscosity is a familiar notion in a field theory, but does it have a direct interpretation in the gravity side? In fact, physicists studying black holes in the 1970's came up with the notion of a membrane paradigm. The horizon of the black hole, as viewed by an outside observer, appears to have all the properties of a physical membrane. Among other things the membrane has an energy momentum tensor which is that of a fluid. Essentially, this follows from the behaviour of geodesics in the vicinity of the horizon. The shear, rotation *etc.* of these geodesics, as introduced by AKR, are instrumental in the identification with a fluid. The respective coefficents then give the various viscosity coefficents. Amazingly, the ordinary 4d Schwarzschild black hole also has a ratio of $\eta/s = 1/4\pi$ in this membrane paradigm.

It therefore seems that the hydrodynamics of the gauge theory is holographically capturing the membrane description of black hole horizons. We have grown familiar with the notion of associating an entropy to black hole horizons and now understand it as the entropy of the microscopic gauge description of the system. Similarly, we are now viewing the hydrodynamic properties that can be attributed to the horizon as those of the hydrodynamics of the gauge theory. Thus the holographic gauge theory description gives a physical meaning and sharper picture to the ideas of the membrane paradigm.

There is an aspect of this connection that the attentive reader might have found puzzling. We had mentioned earlier that the gauge theory is to be viewed as living on the asymptotic boundary of the AdS spacetime. Whereas we are now connecting its properties with those of the horizon which is a different four dimensional surface in the interior of the spacetime. While we don't understand this sharply enough, it appears that long wavelength phenomena of the gauge theory (such as hydrodynamics) capture the dynamics near the horizon. The boundary description is more appropriate as a UV or microscopic one. This dichotomy is a reflection of the fact that the radial direction in the AdS spacetime plays the role of the energy scale in the gauge theory, with asymptotic infinity being the UV and the deep interior near the horizon being the the IR.

To summarise, the realisation of holography in *AdS* spacetimes has enabled us to get a better handle on many aspects of this miraculous connection. The understanding of hydrodynamic phenomena is a step beyond the conventional black hole thermodynamics and shows us how far reaching the connection between the gauge and gravity theories are. It has already provided qualitative and probably quantitative understanding of the SQGP phase discovered at RHIC. The comprehensive nature of the connection promises to also shed light on far from equilibrium situations such as in black hole formation. This has yet to be explored.

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